

# Normal solution and transport coefficients to the Enskog-Landau kinetic equation for a two-component system of charged hard spheres. The Chapman-Enskog method

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## Abstract

An Enskog-Landau kinetic equation for a many-component system of charged hard spheres is proposed. It has been obtained from the Liouville equation with modified boundary conditions by the method of nonequilibrium statistical operator. On the basis of this equation the normal solutions and transport coefficients such as bulk  $\kappa$  and shear  $\eta$  viscosities, thermal conductivity  $\lambda$ , mutual diffusion  $D^{\alpha\beta}$  and thermal diffusion  $D_T^\alpha$  have been obtained for a binary mixture in the first approximation using the Chapman-Enskog method. Numerical calculations of  $\kappa$ ,  $\eta$ ,  $D^{\alpha\beta}$  and  $D_T^\alpha$  for mixtures Ar-Kr, Ar-Xe, Kr-Xe with different concentrations of compounds have been evaluated for the cases of absence and presence of long-range Coulomb interactions. The results are compared with those obtained from other theories and experiment.

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## 1. Introduction

The Enskog-Landau kinetic equation for an one-component system of charged hard spheres [1] is comparatively new. Being obtained from the first principles of statistical mechanics it describes well transport processes in moderately dense systems. Its collision integral does not have any singularities on short-range distances. Numerical calculations of shear viscosity  $\eta$  and thermal conductivity  $\lambda$  on the basis of normal solutions to this equation using the Chapman-Enskog method are in a good agreement between theory and experiment [2]. In next papers [3, 4], nonstationary solutions and transport coefficients have been obtained using the boundary conditions method. In the limiting case of stationary processes, the results of [2] and [4] coincide between themselves. Despite this fact, the one-component system remains always by a model system [6, 7, 8].

Traditional interest in many-component systems, including two-component mixtures of dense gases, liquids and plasmas, has been intensified recently by the considerable advancement in computer equipment. This allows to supplement theory by results of mathematical simulations rather easily.

The transport coefficients of nonequilibrium systems have been investigated using different methods, namely: Bhatnagar-Gross-Kruk theory [9], Green-Kubo [10] and Frost [11] methods, at unified description of kinetics and equations of nonlinear hydrodynamics [12], in one-liquid approximation [13]. Mathematical construction of extended thermodynamics of dense gases and liquids (including hard sphere mixtures within RET theory [1]) has been carried out [14], thermodynamical compatibility of Enskog-type kinetic equations for  $M$ -component systems and Enskog-like kinetic equations for reacting mixtures has been proved [15], results of macroscopic and microscopic theories have been compared [16]. At the same time, when methods for calculating transport coefficients were improved, numerical simulations such as Monte Carlo (MC) and molecular dynamics (MD) were optimized [17, 18, 19] also. In some cases, purely theoretical problems lead to some other ones: the Gross-Jackson model [20], inverse problem of thermal conductivity [21], effective transport coefficients [22, 23, 24]. In such a way the efforts to expand the existing theory [1, 2] on many-component systems looks quite naturally. Moreover, a sequential theory for charged particles does not exist yet. In papers [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24], transport coefficients (such as viscosity  $\eta$ , conductivity  $\sigma$ , thermal conductivity  $\lambda$ , mutual diffusion  $D^{\alpha\beta}$  and thermal diffusion  $D_T^\alpha$ ) have been calculated in one or other way. It is pointed everywhere, that the best coincidence of theoretical and experimental data (as well as MC or MD) is obtained for neutral systems at comparatively not high densities. As was noted in [18], the coincidence at high densities is bad (theory does not work). It is summarized in [19] on the basis of MC simulations, that  $\eta$  coincides with experimental value better than  $\lambda$  in major cases. Generally, a worse agreement of theory and experiment for  $\lambda$  is known for a long time. First of all, it is connected with a strong growing of interparticle correlations as density decreases. These correlations should be taken into account more precisely [25]. In its turn, this leads to changes in local conservation laws, which are used at finding for solutions to kinetic equations [4].

In the present paper, the Enskog-Landau kinetic equation for a multicomponent system of charged hard spheres, its normal solution by means of the Chapman-Enskog method, and the calculation of transport coefficients for binary mixtures are proposed. The theoretical part of this work was published for the first time in papers [26, 27]. Now it is upgraded by some results of numerical calculations for transport coefficients as well as their comparison with some experiments and MD data. A special attention is for  $\Omega$ -integrals calculation for long-range type of interaction (Coulomb interaction in our case) and for finding of static screening radius. Expressions for spatially dependent distribution functions in relations for transport coefficients are discussed concisely as well.

## 2. Initial relations

Let us consider a nonequilibrium system of  $N$  classical particles, which consists of  $M$  kinds of charged hard spheres. Each kind has  $N_\alpha$  particles, where  $\alpha$  is varied between 1 and  $M$ , with  $\sum_{\alpha=1}^M N_\alpha = N$ . The masses of particles are equal to  $m_\alpha$ , altogether they occupy a volume  $V$ . It is agreed that the thermodynamic limit transition  $N \rightarrow \infty$ ,  $V \rightarrow \infty$ ,  $N/V = \text{const}$  takes place. The charges of particles are equal to  $Z_\alpha e$ , where  $e$  is the electron charge,  $Z_\alpha \in \mathbb{Z}$ . The condition of electroneutrality is applied to the system as a whole. It means that in the case of differently charged particles  $\sum_{\alpha=1}^M N_\alpha Z_\alpha = 0$ , while in the opposite cases, where only positively charged particles are considered (mixtures of ionized gases, for example), a neutralizing continuum is introduced.

The Hamiltonian of such a system can be presented as follows:

$$H = \sum_{\alpha=1}^M \sum_{j_\alpha=1}^{N_\alpha} \frac{p_{j_\alpha}^2}{2m_\alpha} + \frac{1}{2} \sum_{\alpha=1}^M \sum_{\beta=1}^M \sum_{j_\alpha=1}^{N_\alpha} \sum_{k_\beta=1}^{N_\beta} \Phi(|\mathbf{r}_{j_\alpha} - \mathbf{r}_{k_\beta}|),$$

$j_\alpha \neq k_\beta \text{ at } \alpha=\beta$

where

$$\Phi(|\mathbf{r}_{j_\alpha} - \mathbf{r}_{k_\beta}|) = \Phi(|\mathbf{r}_{j_\alpha k_\beta}|) = \Phi^{\text{hs}}(|\mathbf{r}_{j_\alpha k_\beta}|) + \Phi^{\text{l}}(|\mathbf{r}_{j_\alpha k_\beta}|)$$

is the total interparticle interaction potential, which consists of the short-range (hard spheres)  $\Phi^{\text{hs}}$  and the long-range (Coulomb one in our case)  $\Phi^{\text{l}}$  interactions:

$$\Phi^{\text{hs}}(|\mathbf{r}_{j_\alpha k_\beta}|) = \lim_{c \rightarrow \infty} \Phi^c(|\mathbf{r}_{j_\alpha k_\beta}|) = \begin{cases} c, & |\mathbf{r}_{j_\alpha k_\beta}| < \sigma_{\alpha\beta}, \\ 0, & |\mathbf{r}_{j_\alpha k_\beta}| > \sigma_{\alpha\beta}, \end{cases}$$

$$\Phi^{\text{l}}(|\mathbf{r}_{j_\alpha k_\beta}|) = \begin{cases} 0, & |\mathbf{r}_{j_\alpha k_\beta}| < \sigma_{\alpha\beta}, \\ \frac{Z_\alpha Z_\beta e^2}{|\mathbf{r}_{j_\alpha k_\beta}|}, & |\mathbf{r}_{j_\alpha k_\beta}| > \sigma_{\alpha\beta}. \end{cases}$$

A nonequilibrium state of this system is described by the nonequilibrium  $N$ -particle distribution function  $\varrho(x^N; t)$ . Let us consider now much precisely the case of a two-component system, when  $M = 2$ . Then the nonequilibrium  $N$ -particle distribution function,

$$\begin{aligned} \varrho(x^N; t) &= \varrho((x^a)^{N_a}, (x^b)^{N_b}; t) \\ &= \varrho(x_1^a, x_2^a, \dots, x_{N_a}^a, x_1^b, x_2^b, \dots, x_{N_b}^b; t), \end{aligned}$$

satisfies the Liouville equation and normalization condition [1, 2]. In the two-component case the  $N$ -particle Liouville operator has a different structure, which takes into account a many-component nature of the system:

$$L_N = \sum_{\alpha=1}^M \sum_{j_\alpha=1}^{N_\alpha} L(j_\alpha) + \frac{1}{2} \sum_{\alpha=1}^M \sum_{\beta=1}^M \sum_{j_\alpha=1}^{N_\alpha} \sum_{k_\beta=1}^{N_\beta} L(j_\alpha, k_\beta),$$

$j_\alpha \neq k_\beta \text{ at } \alpha=\beta$

$$L(j_\alpha) = -i \frac{\mathbf{p}_{j_\alpha}}{m_\alpha} \frac{\partial}{\partial \mathbf{r}_{j_\alpha}},$$

$$L(j_\alpha, k_\beta) = i \frac{\partial \Phi(|\mathbf{r}_{j_\alpha} - \mathbf{r}_{k_\beta}|)}{\partial \mathbf{r}_{j_\alpha}} \left( \frac{\partial}{\partial \mathbf{p}_{j_\alpha}} - \frac{\partial}{\partial \mathbf{p}_{k_\beta}} \right).$$

### 3. Kinetic equation

The kinetic equation for  $\alpha$ -kind one-particle distribution function  $f_1(x_1^\alpha; t)$  can be obtained similarly to an one-component system, but with some distinguishes, which are due to a multicomponent nature of the system under consideration. In such a way, in the “pair” collisions approximation this equation reads:

$$\left( \frac{\partial}{\partial t} + iL(1_\alpha) \right) f_1(x_1^\alpha; t) = - \sum_{\beta=1}^M \int dx_2^\beta iL(1_\alpha, 2_\beta) \times \lim_{\tau \rightarrow -\infty} e^{iL_2 \tau} g_2(\mathbf{r}_1^\alpha, \mathbf{r}_2^\beta; t + \tau) f_1(x_1^\alpha; t + \tau) f_1(x_2^\beta; t + \tau), \quad (3.1)$$

where

$$g_2(\mathbf{r}_1^\alpha, \mathbf{r}_2^\beta; t) = [n(\mathbf{r}_1^\alpha; t) n(\mathbf{r}_2^\beta; t)]^{-1} f_2(\mathbf{r}_1^\alpha, \mathbf{r}_2^\beta; t),$$

$$f_2(\mathbf{r}_1^\alpha, \mathbf{r}_2^\beta; t) = \int d\Gamma'_N n_2(\mathbf{r}_1^\alpha, \mathbf{r}_2^\beta(x')^N; t) \varrho_q((x')^N; t).$$

Equation (3.1) is formally distinguished on analogous one for an one-component system by the presence of kind summations on its right-hand part in the collision integral. The Enskog-Landau kinetic equation for a two-component system of charged hard spheres has been obtained for the first time in [26]:

$$\left[ \frac{\partial}{\partial t} + iL(1_\alpha) \right] f_1(x_1^\alpha; t) = I_E^{(0)}(x_1^\alpha; t) + I_E^{(1)}(x_1^\alpha; t) + I_{MF}(x_1^\alpha; t) + I_L(x_1^\alpha; t), \quad (3.2)$$

where terms in the right-hand part are collision integrals. Explicit expressions for the collision integrals depend on a specific interparticle potential of interaction. The first and the second terms are collision integrals of the revised Enskog theory (RET) [1, 2]:

$$I_E^{(0)}(x_1^\alpha; t) = \sum_{\beta=1}^M \int d\mathbf{v}_2^\beta d\varepsilon db b g^{\alpha\beta} g_2^{\alpha\beta}(\sigma_{\alpha\beta}|n, \beta) \times \left( f_1(\mathbf{r}_1^\alpha, \mathbf{v}_1^{\alpha'}; t) f_1(\mathbf{r}_2^\beta, \mathbf{v}_2^{\beta'}; t) - f_1(\mathbf{r}_1^\alpha, \mathbf{v}_1^\alpha; t) f_1(\mathbf{r}_2^\beta, \mathbf{v}_2^\beta; t) \right), \quad (3.3)$$

$$I_E^{(1)}(x_1^\alpha; t) = \sum_{\beta=1}^M \sigma_{\alpha\beta}^3 \int d\hat{\mathbf{r}}_{12}^{\alpha\beta} d\mathbf{v}_2^\beta \Theta(\hat{\mathbf{r}}_{12}^{\alpha\beta} \cdot \mathbf{g}^{\alpha\beta}) (\hat{\mathbf{r}}_{12}^{\alpha\beta} \cdot \mathbf{g}^{\alpha\beta}) \times$$

$$\begin{aligned}
& \left[ \hat{\mathbf{r}}_{12}^{\alpha\beta} g_2^{\alpha\beta}(\mathbf{r}_{12}^{\alpha\beta} | n, \beta) \times \right. \\
& \left[ f_1(\mathbf{r}_1^\alpha, \mathbf{v}_1^{\alpha'}; t) \nabla f_1(\mathbf{r}_2^\beta, \mathbf{v}_2^{\beta'}; t) - f_1(\mathbf{r}_1^\alpha, \mathbf{v}_1^\alpha; t) \nabla f_1(\mathbf{r}_2^\beta, \mathbf{v}_2^\beta; t) \right] + \\
& \frac{1}{2} \left[ \hat{\mathbf{r}}_{12}^{\alpha\beta} \cdot \nabla g_2^{\alpha\beta}(\mathbf{r}_{12}^{\alpha\beta} | n, \beta) \right] \times \\
& \left. \left[ f_1(\mathbf{r}_1^\alpha, \mathbf{v}_1^{\alpha'}; t) f_1(\mathbf{r}_2^\beta, \mathbf{v}_2^{\beta'}; t) + f_1(\mathbf{r}_1^\alpha, \mathbf{v}_1^\alpha; t) f_1(\mathbf{r}_2^\beta, \mathbf{v}_2^\beta; t) \right] \right].
\end{aligned} \tag{3.4}$$

The next term is caused by the influence of long-range interactions in the mean field approximation (KMFT) [1, 2]:

$$\begin{aligned}
I_{\text{MF}}(x_1^\alpha; t) = \\
\frac{1}{m_\alpha} \sum_{\beta=1}^M \int d\mathbf{r}_2^\beta \frac{\partial}{\partial \mathbf{r}_1^\alpha} \Phi^1(|\mathbf{r}_{12}^{\alpha\beta}|) g_2^{\alpha\beta}(\mathbf{r}_1^\alpha, \mathbf{r}_2^\beta; t) \frac{\partial}{\partial \mathbf{v}_1^\alpha} f_1(x_1^\alpha; t) n_\beta(\mathbf{r}_2^\beta; t),
\end{aligned} \tag{3.5}$$

the last term is a Landau-like collision integral [1, 2]:

$$\begin{aligned}
I_{\text{L}}(x_1^\alpha; t) = \sum_{\beta=1}^M \int d\mathbf{v}_2^\beta d\varepsilon db b g^{\alpha\beta} \times \\
\left( f_1(\mathbf{r}_1^\alpha, \mathbf{v}_1^{\alpha*}; t) f_1(\mathbf{r}_2^\beta, \mathbf{v}_2^{\beta*}; t) - f_1(\mathbf{r}_1^\alpha, \mathbf{v}_1^\alpha; t) f_1(\mathbf{r}_2^\beta, \mathbf{v}_2^\beta; t) \right).
\end{aligned} \tag{3.6}$$

This collision integral is presented here in a reduced Boltzmann-like form. It could be obtained going from the cartesian to cylindrical reference frame, entering impact parameter  $b$ , azimuthal angle of scattering  $\varepsilon$ , distance along cylinder centreline  $\xi$  and integrating with respect to  $\xi$  and taking into account the condition  $g_2(\mathbf{r}_1, \mathbf{r}_2; t) \rightarrow 1$ . If one solves this kinetic equation using the Chapman-Enskog method, this form for the latest collision integral is more convenient [2]. Other conventional designations in (3.3) – (3.6) are as follows:

$b$	impact parameter,
$\beta$	inversed local temperature analogue,
$\varepsilon$	azimuthal angle of scattering,
$\mathbf{g}^{\alpha\beta}$	relative velocity of $\alpha$ - and $\beta$ -kind particles,
$g_2^{\alpha\beta}$	two-particle correlation function,
$m^*$	reduced mass,
$m_\alpha$	partial masses of particles,
$n$	total density of particles number,
$n_\alpha$	partial densities of particles numbers,
$\hat{\mathbf{r}}_{12}^{\alpha\beta}$	unit vector along $\mathbf{r}_{12}^{\alpha\beta}$ direction,
$\Theta(x)$	unit step function.

An additional point for emphasizing is that  $\mathbf{v}_1^{\alpha'}$  and  $\mathbf{v}_2^{\beta'}$  are hard spheres velocities after collision:

$$\begin{aligned}
\mathbf{v}_1^{\alpha'} &= \mathbf{v}_1^\alpha + \hat{\mathbf{r}}_{12}^{\alpha\beta} (\hat{\mathbf{r}}_{12}^{\alpha\beta} \cdot \mathbf{g}^{\alpha\beta}), \\
\mathbf{v}_2^{\beta'} &= \mathbf{v}_2^\beta - \hat{\mathbf{r}}_{12}^{\alpha\beta} (\hat{\mathbf{r}}_{12}^{\alpha\beta} \cdot \mathbf{g}^{\alpha\beta}),
\end{aligned}$$

as well as  $\mathbf{v}_1^{\alpha*}$  and  $\mathbf{v}_2^{\alpha*}$  are velocities of charged particles after Coulomb scattering:

$$\begin{aligned} \mathbf{v}_1^{\alpha*} &= \mathbf{v}_1^\alpha + \Delta \mathbf{v}^{\alpha\beta}, \\ \mathbf{v}_2^{\alpha*} &= \mathbf{v}_1^\beta - \Delta \mathbf{v}^{\alpha\beta}, \\ \Delta \mathbf{v}^{\alpha\beta} &= -\frac{1}{m^*} \int d\xi \frac{\partial}{\partial \mathbf{r}_{12}^{\alpha\beta}} \Phi^1(|\mathbf{r}_{12}^{\alpha\beta}|) \frac{1}{g^{\alpha\beta}} \bigg|_{r_{12}^{\alpha\beta} = \sqrt{b^2 + \xi^2}}. \end{aligned} \quad (3.7)$$

#### 4. Normal solutions by means of the Chapman-Enskog method in the first approximation

As usually, to solve kinetic equation by means of the Chapman-Enskog method in the  $k$ th approximation we will use conservation laws for a set of hydrodynamical variables in the  $(k-1)$ th approximation. The components of the additive invariant vector could be chosen similarly to [2, 4], namely the hydrodynamical mass density, momentum and energy:

$$\begin{aligned} \rho(\mathbf{r}_1; t) &= \sum_{\alpha=1}^M \int d\mathbf{v}_1^\alpha f_1(x_1^\alpha; t) m_\alpha, \\ \rho(\mathbf{r}_1; t) \mathbf{V}(\mathbf{r}_1; t) &= \sum_{\alpha=1}^M \int d\mathbf{v}_1^\alpha f_1(x_1^\alpha; t) m_\alpha \mathbf{v}_1^\alpha, \\ \rho(\mathbf{r}_1; t) \omega_k(\mathbf{r}_1; t) &= \sum_{\alpha=1}^M \int d\mathbf{v}_1^\alpha f_1(x_1^\alpha; t) \frac{m_\alpha (\mathbf{c}_1^\alpha)^2}{2}. \end{aligned}$$

The conservation law for  $\rho(\mathbf{r}_1; t)$  have the form of a continuity equation, whereas the equation of motion and equation of energy balance read as

$$\begin{aligned} \rho(\mathbf{r}_1; t) \frac{d\mathbf{V}(\mathbf{r}_1; t)}{dt} &= -\frac{\partial}{\partial \mathbf{r}_1} \vec{P}'(\mathbf{r}_1; t), \\ \rho(\mathbf{r}_1; t) \frac{d\omega_k(\mathbf{r}_1; t)}{dt} &= -\frac{\partial}{\partial \mathbf{r}_1} \mathbf{q}^*(\mathbf{r}_1; t) - \vec{P}^*(\mathbf{r}_1; t) : \frac{\partial}{\partial \mathbf{r}_1} \mathbf{V}(\mathbf{r}_1; t) + \\ &\quad \sum_{\alpha, \beta}^M \mathbf{V}_d^\alpha(\mathbf{r}_1; t) \int d\mathbf{r}_{12}^{\alpha\beta} \frac{\partial}{\partial r_{12}^{\alpha\beta}} \Phi^1(|\mathbf{r}_{12}^{\alpha\beta}|) n_\alpha(\mathbf{r}_1^\alpha; t) n_\beta(\mathbf{r}_1^\alpha + \mathbf{r}_{12}^{\alpha\beta}; t), \end{aligned} \quad (4.1)$$

where

$$\begin{aligned} \vec{P}'(\mathbf{r}_1; t) &= \vec{P}^k(\mathbf{r}_1; t) + \vec{P}^{\text{hs}}(\mathbf{r}_1; t) + \vec{P}^{\text{mf}}(\mathbf{r}_1; t), \\ \vec{P}^*(\mathbf{r}_1; t) &= \vec{P}^k(\mathbf{r}_1; t) + \vec{P}^{\text{hs}}(\mathbf{r}_1; t), \\ \mathbf{q}^*(\mathbf{r}_1; t) &= \mathbf{q}^k(\mathbf{r}_1; t) + \mathbf{q}^{\text{hs}}(\mathbf{r}_1; t); \end{aligned}$$

$\mathbf{V}_d^\alpha(\mathbf{r}_1; t)$  is the diffusion velocity of  $\alpha$ -kind particles:

$$\mathbf{V}_d^\alpha(\mathbf{r}_1; t) = \langle \mathbf{c}^\alpha(\mathbf{r}_1; t) \rangle, \quad \mathbf{c}^\alpha(\mathbf{r}_1; t) = \mathbf{v}_1^\alpha - \mathbf{V}(\mathbf{r}_1; t),$$

here  $\langle \dots \rangle$  denotes an average value over the one-particle distribution function  $f_1(x_1^\alpha; t)$ . In the zeroth approximation, when the function  $f_1(x_1^\alpha; t)$  is taken as a local-equilibrium Maxwell distribution function ( $k$  is the Boltzmann constant),

$$f_1^{(0)}(x_1^\alpha; t) = n_\alpha(\mathbf{r}_1^\alpha; t) \left( \frac{m_\alpha}{2\pi kT(\mathbf{r}_1^\alpha; t)} \right)^{3/2} \exp \left\{ -\frac{m_\alpha (c_1^\alpha(\mathbf{r}_1^\alpha; t))^2}{2kT(\mathbf{r}_1^\alpha; t)} \right\},$$

one obtains for  $\vec{P}'(\mathbf{r}_1; t)$ ,  $\mathbf{q}^*(\mathbf{r}_1; t)$ ,  $\mathbf{V}_d^\alpha(\mathbf{r}_1; t)$  the following expressions:

$$\vec{P}^k = \vec{I} P^k, \quad P^k = \sum_{\alpha=1}^M n_\alpha kT, \quad (4.2)$$

$$\vec{P}^{\text{hs}} = \vec{I} P^{\text{hs}}, \quad P^{\text{hs}} = \frac{2}{3} \pi kT \sum_{\alpha, \beta}^M n_\alpha n_\beta \sigma_{\alpha\beta}^3 g_2^{\alpha\beta}(\sigma|n, \beta),$$

$$\vec{P}^{\text{mf}} = \vec{I} P^{\text{mf}}, \quad P^{\text{mf}} = -\frac{2}{3} \pi \sum_{\alpha, \beta}^M n_\alpha n_\beta \int_{\sigma_{\alpha\beta}}^{\infty} dx x^3 (\Phi_{\alpha\beta}^1(x))' g_2^{\alpha\beta}(x),$$

$$\vec{P}^1 = 0, \quad \mathbf{q}^k = \mathbf{q}^{\text{hs}} = \mathbf{q}^{\text{mf}} = \mathbf{q}^1 = 0, \quad \mathbf{V}_d^\alpha(\mathbf{r}_1; t) = 0.$$

Using the mass conservation law and equation (4.1) in the zeroth approximation and taking into consideration an explicit form for  $\vec{P}'(\mathbf{r}_1; t)$ ,  $\vec{P}^*(\mathbf{r}_1; t)$ ,  $\mathbf{q}^*(\mathbf{r}_1; t)$  (4.2), we will find for the solution to equation (3.2) in the first approximation as the combination:

$$f_1(x_1^\alpha; t) = f_1^{(0)}(x_1^\alpha; t) (1 + \varphi(x_1^\alpha; t)), \quad (4.3)$$

where the correction  $\varphi(x_1^\alpha; t)$  is expressed through the Sonine-Laguerre polynomials:

$$\begin{aligned} \varphi(x_1^\alpha; t) &= \sqrt{\frac{m_\alpha}{2kT}} E^\alpha \left( \frac{m_\alpha (c_1^\alpha)^2}{2kT} \right) n(c_1^\alpha, \mathbf{d}^\alpha) - \\ &\quad \sqrt{\frac{m_\alpha}{2kT}} A^\alpha \left( \frac{m_\alpha (c_1^\alpha)^2}{2kT} \right) (c_1^\alpha, \nabla) \ln T - \\ &\quad \frac{m_\alpha}{2kT} B^\alpha \left( \frac{m_\alpha (c_1^\alpha)^2}{2kT} \right) \left( c_1^\alpha c_1^\alpha - \frac{1}{3} (c_1^\alpha)^2 \vec{I} \right) : \nabla \mathbf{V}. \end{aligned}$$

Here

$$\begin{aligned} E^\alpha(x) &= \sum_{n=0}^{\infty} E_n^\alpha L_n^{3/2}(x), \\ A^\alpha(x) &= \sum_{n=0}^{\infty} A_n^\alpha L_n^{3/2}(x), \quad B^\alpha(x) = \sum_{n=0}^{\infty} B_n^\alpha L_n^{5/2}(x), \\ L_n^r(x) &= \sum_{m=0}^n (-1)^m x^m \frac{n! \Gamma(n+r+1)}{m! \Gamma(m+r+1) \Gamma(n-m+1)}, \end{aligned}$$

$\mathbf{d}^\alpha \equiv \mathbf{d}^\alpha(\mathbf{r}_1^\alpha; t)$  is the diffusion thermodynamic force of  $\alpha$ -kind of mixture [28]. Quantities  $\mathbf{d}^\alpha$  satisfy the condition

$$\sum_{\alpha=1}^M \mathbf{d}^\alpha(\mathbf{r}_1^\alpha; t) = 0.$$

In the case of an one-component system, diffusion thermodynamic forces are absent.

If one puts the solution of (4.3) into equation (3.2), one shall obtain a nonuniform integral equation of Fredholm-type. Therefore, additional conditions for the existence of solutions to this equation can be written down at once: this is orthogonality of the right-hand side of (3.2) up to solutions of the corresponding uniform equation. The last ones are parameters of the abbreviated hydrodynamic description. Whereas, additional conditions after mathematical transformations read:

$$E_0^b = -\frac{n_a}{n_b} \sqrt{\frac{m_a}{m_b}} E_0^a, \quad A_0^b = -\frac{n_a}{n_b} \sqrt{\frac{m_a}{m_b}} A_0^a.$$

This should be taken into consideration any time one looks for expansion coefficients  $E_n^\alpha$ ,  $A_n^\alpha$ ,  $B_n^\alpha$ . In such a way, several independent infinite systems of equations appear. It is impossible, of course, to find an exact solution to these systems. One should restrict by a few first terms in expansion series. As a rule, practically, one considers quantities  $E_0^\alpha$ ,  $B_0^\alpha$  and  $A_0^\alpha$ ,  $A_1^\alpha$  only. We should take in mind the fact, that the collision of identical particles gives no affect due to mass, momentum and energy conservation laws at elastic scattering.

The basic relations to obtain  $A_0^\alpha$  and  $A_1^\alpha$  coefficients as well as their symbolic solution are presented in Appendix A. The same for  $B_0^\alpha$  coefficients is done in Appendix B.

## 5. Transport coefficients

Having one-particle distribution function (4.3) in the first approximation, we are able to write down the expression for mass density flow of  $\alpha$ -component of mixture [29]:

$$\mathbf{j}^\alpha(\mathbf{r}_1^\alpha; t) = -\rho^{-1} n^2 m_\alpha m_\beta D^{\alpha\beta} \mathbf{d}^\alpha(\mathbf{r}_1^\alpha; t) - D_T^\alpha \nabla \ln T(\mathbf{r}_1^\alpha; t),$$

where  $D^{\alpha\beta}$  is the mutual diffusion coefficient, and  $D_T^\alpha$  is the thermal diffusion coefficient of  $\alpha$ -kind of mixture. In the zeroth polynomial approximation

$$D^{\alpha\beta} = -\frac{n_\alpha \rho}{m_\beta n} \sqrt{\frac{kT}{2m_\alpha}} E_0^\alpha, \quad (5.1)$$

$$D_T^\alpha = m_\alpha n_\alpha \sqrt{\frac{kT}{2m_\alpha}} A_0^\alpha. \quad (5.2)$$

Then the expression for thermal diffusion ratio is easy to write as

$$k_T = \frac{\rho}{n^2 m_\alpha m_\beta} \frac{D_T}{D^{\alpha\beta}} = -\frac{1}{n} \frac{A_0^\alpha}{E_0^\alpha}.$$



Final expressions for coefficients of thermal diffusion one can write taking into account the structure of  $A_0^\alpha$ -coefficients (Appendix A.). To calculate quantities  $E_0^\alpha$  it is convenient to consider new variables, namely, the variables of inertia centre. By that transformations these quantities are calculated exactly. This allows to write the expression for mutual diffusion coefficient in an analytical form:

$$\begin{aligned} E_0^\alpha &= -\frac{3\pi m_\beta}{8\rho n_\alpha} \sqrt{\frac{\pi m_\alpha}{m^*}} \left( g_2^{\alpha\beta}(\sigma_{\alpha\beta}|n, \beta) {}^{\alpha\beta}\Omega_{\text{hs}}^{(1,1)} + {}^{\alpha\beta}\Omega_1^{(1,1)} \right)^{-1}, \\ D^{\alpha\beta} &= \frac{3\pi}{8n} \sqrt{\frac{\pi kT}{2m^*}} \left( g_2^{\alpha\beta}(\sigma_{\alpha\beta}|n, \beta) {}^{\alpha\beta}\Omega_{\text{hs}}^{(1,1)} + {}^{\alpha\beta}\Omega_1^{(1,1)} \right)^{-1}. \end{aligned}$$

Here  ${}^{\alpha\beta}\Omega_{\text{hs}}^{(r,p)}$ ,  ${}^{\alpha\beta}\Omega_1^{(r,p)}$  are known as so-called  $\Omega$ -integrals [30]. The  $\Omega$ -integrals appear in calculation of collision integrals (integral parentheses [30]) in kinetic theory of gases, liquids and plasmas. The explicit form of  $\Omega$ -integrals depends on type of interparticle interaction, that is taken into consideration. In our case, the total interparticle interaction potential consists on hard sphere and Coulomb parts. In such a way, two types of  $\Omega$ -integrals will appear,

$$\begin{aligned} {}^{\alpha\beta}\Omega_{\text{hs}}^{(r,p)} &= \int_0^\infty dy e^{-y^2} y^{2p+3} {}^{\alpha\beta}\Omega_{\text{hs}}^{(r)}, \\ {}^{\alpha\beta}\Omega_1^{(r,p)} &= \int_0^\infty dy e^{-y^2} y^{2p+3} {}^{\alpha\beta}\Omega_1^{(r)}, \\ {}^{\alpha\beta}\Omega_{\text{hs}}^{(r)} &= 2\pi \int_0^{\sigma_{\alpha\beta}} db b \left[ 1 - \cos^r \left( {}^{\alpha\beta}\chi'(b, y) \right) \right], \\ {}^{\alpha\beta}\Omega_1^{(r)} &= 2\pi \int_{\sigma_{\alpha\beta}}^\infty db b \left[ 1 - \cos^r \left( {}^{\alpha\beta}\chi^*(b, y) \right) \right]. \end{aligned}$$

Here  $\chi'$  is the scattering angle of hard spheres after collision,  $\chi^*$  is the scattering angle of charged particles. The analytical calculation of  $\Omega$ -integrals presented in Appendix C.

Except the thermal and mutual diffusion coefficients in the first approximation, the stress tensor  $\vec{P}'(\mathbf{r}_1; t)$  and heat flow vector  $\mathbf{q}^*(\mathbf{r}_1; t)$  one can calculate also and obtain other transport coefficients. The calculations give:

$$\vec{P}(\mathbf{r}_1; t) = P(\mathbf{r}_1; t) \vec{I} - \kappa(\nabla : \mathbf{V}(\mathbf{r}_1; t)) - 2\eta \vec{S}(\mathbf{r}_1; t),$$

$$\mathbf{q}(\mathbf{r}_1; t) = -\lambda \nabla T(\mathbf{r}_1; t) + \sum_{\alpha=1}^M \omega_\alpha \mathbf{d}^\alpha.$$

Here  $\kappa$  is the total bulk viscosity coefficient of mixture:

$$\kappa = \frac{8}{9} \sum_{\alpha, \beta=1}^M \sigma_{\alpha\beta}^4 g_2^{\alpha\beta}(\sigma_{\alpha\beta}|n, \beta) n_\alpha n_\beta \frac{m^*}{m_\beta} \sqrt{2\pi m^* kT} = \sum_{\alpha, \beta=1}^M \kappa_{\alpha\beta}, \quad (5.3)$$

$\eta$  is the total shear viscosity coefficient:

$$\eta = \frac{3}{5}\kappa + \frac{1}{2} \sum_{\alpha=1}^M n_{\alpha} kT \times \left[ 1 + \frac{2\pi}{15} \sum_{\beta=1}^M n_{\beta} \sigma_{\alpha\beta}^3 g_2^{\alpha\beta}(\sigma_{\alpha\beta}|n, \beta) \left[ 1 + \frac{m_{\alpha} B_0^{\beta}}{m_{\beta} B_0^{\alpha}} \right] \right] B_0^{\alpha}, \quad (5.4)$$

$\lambda$  is the total thermal conductivity coefficient:

$$\begin{aligned} \lambda = & \frac{3}{2} \sum_{\alpha, \beta=1}^M k m_{\alpha} m_{\beta} \left[ \frac{1}{2}(m_{\alpha} + m_{\beta}) - \frac{1}{8} \frac{(m_{\alpha} - m_{\beta})^2}{m_{\alpha} + m_{\beta}} \right]^{-3} \kappa_{\alpha\beta} + \\ & \frac{5}{4} \sum_{\alpha=1}^M n_{\alpha} k \sqrt{\frac{2kT}{m_{\alpha}}} \times \\ & \left[ 1 - \frac{A_0^{\alpha}}{A_1^{\alpha}} + \frac{\pi}{5} \sum_{\beta=1}^M n_{\beta} \sigma_{\alpha\beta}^3 g_2^{\alpha\beta}(\sigma_{\alpha\beta}|n, \beta) \left[ 1 + \frac{m_{\alpha}^{3/2} A_1^{\beta}}{m_{\beta}^{3/2} A_1^{\alpha}} \right] \right] A_1^{\alpha}, \\ \omega_{\alpha} = & \frac{5}{4} n n_{\alpha} \sqrt{\frac{2k^3 T^3}{m_{\alpha}}} E_0^{\alpha}. \end{aligned} \quad (5.5)$$

There is important to consider transport coefficients (5.1) – (5.5) in a limiting transition, when the number of kinds  $M$  goes to unity. In such a case one must obtain results of theory for an one-component system of charged hard spheres in a neutralizing background [2]. This is easy to see if one puts

$$\begin{aligned} m_a = m_b = m, & \quad E_0^{\alpha} = A_0^{\alpha} = 0, \\ n_a = n_b = n/2, & \quad B_0^{\alpha} = B_0^{\beta} = B_0. \end{aligned}$$

By this means, there are no diffusion and thermal diffusion coefficients in an one-component system due to the absence of thermodynamic diffusion forces  $\mathbf{d}^{\alpha}$ . But the equality  $A_0^{\alpha} = 0$  appears as the Fredholm condition.

## 6. Numerical calculations

In our paper numerical calculations have been carried out for transport coefficients  $D^{\alpha\beta}$  (5.1),  $D_T^{\alpha}$  (5.2),  $\kappa$  (5.3) and  $\eta$  (5.4). We have chosen mixtures of Ar-Kr, Ar-Xe, Kr-Xe as objects of our calculations. Neutral and charged mixtures of these gases are appropriate well for the model of neutral and charged hard spheres, which is considered here. This is so because other mixtures can have complex molecular structures. The transport coefficients have been calculated at several fixed dimensionless densities  $\Delta$  in a wide physically real for these mixtures ranges of temperature ( $\Delta = \sum_{\alpha=1}^M \Delta_{\alpha} \sigma_{\alpha}^3$ ,  $\Delta_{\alpha} = \pi n_{\alpha}/6$ ,  $\sigma$  is a hard sphere diameter).

Magnitudes for neutral systems ( $Z_{\alpha} = 0$ ) of some transport coefficients ( $D^{\alpha\beta}$ ,  $\eta$ ) of Ar-Kr mixture have been compared with results of MD [31, 32] and HSED

theory [33] for two fixed temperature points  $T_1 = 508$  K and  $T_2 = 1196$  K at the same dimensionless density  $\Delta = 0.1$  ( $n_\alpha = n_\beta$  of course). The coincidence of these values is exact (see Table 1).

Table 1: The comparison of theoretical calculations of total shear viscosity  $\eta$  and mutual diffusion  $D^{\alpha\beta}$  with results of molecular dynamics simulations and other theories for Lenard-Jones liquids of argon and krypton. \* – data was borrowed from [32], \*\* – data was borrowed from [31], \*\*\* – data was borrowed from [33].

$\Delta$	$T$ , K	$\eta$ , Pa·s, MD	$\eta$ , Pa·s, EDHST	$\eta$ , Pa·s	$D^{\text{Ar-Kr}}$ , $\frac{\text{m}^2}{\text{s}}$ , MD	$D^{\text{Ar-Kr}}$ , $\frac{\text{m}^2}{\text{s}}$
0.1	508	$6.47 \cdot 10^{-5}$ *		$2.58 \cdot 10^{-5}$	$1.518 \cdot 10^{-7}$ *	$2.568 \cdot 10^{-7}$
0.1	1196	$7.72 \cdot 10^{-5}$ **	$8.56 \cdot 10^{-5}$ ***	$3.95 \cdot 10^{-5}$		

Values of shear viscosity of argon component  $\eta_{\text{Ar}}$  of neutral mixture have been compared with results of [34, 35, 36, 37]. Theory parameters  $\sigma_\alpha$  were taken from [38, 39, 40, 41, 42]. The obtained results are in a good agreement with [34, 35, 36, 37, 38, 39, 40, 41, 42] data. All obtained temperature dependencies of *all* transport coefficients at the absence and presence of long-range Coulomb interparticle interactions were published in our recent preprint [48]. But in the present paper we give the combination of some numbers of them only, which reflect good a general tendency. The difference up to two orders in magnitudes for the values of the presented transport coefficients is explained as follows: firstly, different kind of interactions; secondly, different densities. Both these factors affect on magnitudes. This testifies that the obtained formulae are very sensitive on change of system parameters.

Graphs of transport coefficients  $D^{\alpha\beta}$ ,  $D_T^\alpha$  and  $\eta$  in the case of their temperature dependencies are shown in Figure 1, kind concentrations are the same ( $n_\alpha = n_\beta$ ). Each graph is assembled by pairs: neutral mixture ( $Z_\alpha = 0$ ) left and charged mixture ( $Z_\alpha \neq 0$ ) right. This allows to inspect changes in temperature behaviours of transport coefficients visually ( $\sim T^{1/2}$  in Enskog-like theories and  $\sim T^{3/2}$  at presence of long-range Coulomb interparticle interaction). The bulk viscosity  $\kappa$  does not depend on charge  $Z_\alpha$  and  $T$ . In such a way it does not change the nature of its temperature behaviours.

A somewhat other quantitatively pattern appears when  $n_\alpha \neq n_\beta$ . If weighty component of mixture is in greater amount, partial lines in figures become much discrete and vice versa. Moreover, at some correlations of densities they can cross and the influence of light component becomes predominant. But temperature increasing diminishes this difference. (More figures see in details on [www-server:www.icmp.lviv.ua/icmp/preprints/PS/9621Ups.gz](http://www-server:www.icmp.lviv.ua/icmp/preprints/PS/9621Ups.gz)).

At the end of this section one should add a little comment about choosing  $g_2^{\alpha\beta}(r)$  and  $D$ , since they appear in transport coefficient expressions. As have been obtained earlier in this paper, transport coefficients take an analytical structure within the Chapman-Enskog method. But obtaining of any similar relations for

$g_2^{\alpha\beta}(r)$  and  $D$ , which are perceived as being so adapted for considering system closely, is a completely different problem. Thus, in the present paper we have been used results done already. The quantity  $D = 1/2\Gamma$  is chosen as a solution to the next equation:

$$4\Gamma^2 = \frac{e^2}{\varepsilon} \sum_{\alpha=1}^M n_{\alpha} X_{\alpha}^2,$$

$$X_{\alpha} = \left[ Z_{\alpha} - \frac{\pi}{2} \frac{\sigma_{\alpha}^2}{1-\Delta} P_m \right] [1 + \Gamma \sigma_{\alpha}]^{-1},$$

$$P_m = \sum_{\alpha=1}^M \frac{n_{\alpha} \sigma_{\alpha} Z_{\alpha}}{1 + \Gamma \sigma_{\alpha}} \left[ 1 + \frac{\pi}{2(1-\Delta)} \sum_{\alpha=1}^M \frac{n_{\alpha} \sigma_{\alpha}^3}{1 + \Gamma \sigma_{\alpha}} \right]^{-1},$$

which has been proposed in [43]. But the expression for  $g_2^{\alpha\beta}(r)$  has been assembled on the basis of results in [44, 45, 46, 47]. In [44], the Percus-Yevick equation for the radial distribution function in liquid has been generalized to multicomponent mixtures, in [45]  $g_2^{\alpha\beta}(r)$  has been calculated for ion mixtures at the presence of Coulomb and screened Coulomb interactions. A much precise formula for calculation of  $g_2^{\alpha\beta}(r)$  on the basis of the Percus-Yevick approximation has been obtained in [46], as well as a new closure for the Ornstein-Zernike equation for a system of charged particles with specific interparticle interaction potential has been used in [47]. Theoretical solutions there are compared with Monte Carlo simulations. Finally, we conclude that the following structure of  $g_2^{\alpha\beta}(r)$  on hard sphere contact is the most plausible (in the present version of our work we wish to obtain analytical formulae only, choosing actual expressions for  $g_2^{\alpha\beta}$ ):

$$\begin{aligned} g_2^{\alpha\alpha}(\sigma_{\alpha}) &= \left\{ \left( 1 + \frac{\Delta}{2} \right) + \frac{3}{2} \Delta_{\beta} \sigma_{\beta}^2 (\sigma_{\alpha} - \sigma_{\beta}) \right\} (1 - \Delta)^{-2}, \\ g_2^{\beta\beta}(\sigma_{\beta}) &= \left\{ \left( 1 + \frac{\Delta}{2} \right) + \frac{3}{2} \Delta_{\alpha} \sigma_{\alpha}^2 (\sigma_{\beta} - \sigma_{\alpha}) \right\} (1 - \Delta)^{-2}, \\ g_2^{\alpha\beta}(\sigma_{\alpha\beta}) &= \left[ \sigma_{\beta} g_2^{\alpha\alpha}(\sigma_{\alpha}) + \sigma_{\alpha} g_2^{\beta\beta}(\sigma_{\beta}) \right] / 2\sigma_{\alpha\beta}, \\ g_2^{\beta\alpha}(\sigma_{\beta\alpha}) &= \left[ \sigma_{\beta} g_2^{\alpha\alpha}(\sigma_{\alpha}) + \sigma_{\alpha} g_2^{\beta\beta}(\sigma_{\beta}) \right] / 2\sigma_{\alpha\beta}, \\ \sigma_{\alpha\beta} &= \frac{1}{2} (\sigma_{\alpha} + \sigma_{\beta}). \end{aligned}$$

Structurally such a configuration is of simple design also.

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## Appendix A. Calculation of $A_n^\alpha$ -coefficients

The two mutually exchangeable infinite systems of equations for searching  $A_n^\alpha$ -coefficients could be written in the form

$$\sum_{r=0}^{\infty} \left[ \left( \alpha_{sr}^{(a)} + \alpha_{sr}^{(ab,a)} \right) A_r^a + \alpha_{sr}^{(ab,b)} A_r^b \right] = \beta_s^{(a)},$$

$$\sum_{r=0}^{\infty} \left[ \alpha_{sr}^{(ba,a)} A_r^a + \left( \alpha_{sr}^{(b)} + \alpha_{sr}^{(ba,b)} \right) A_r^b \right] = \beta_s^{(b)},$$

where quantities  $\alpha_{sr}$ ,  $\beta_s$  have an integral structure and are expressed via  $L_n^r$  polynomials. General expressions for these quantities are very cumbersome and their forms depend on the structure of collision integrals. But the general form is always the same and is completely shown in [28]. In our case

$$\begin{aligned} \alpha_{sr}^{(a)} = & \int d\mathbf{v}_1^a d\mathbf{v}_2^a g^{aa} d\omega f_1^{a(0)}(\mathbf{c}_1^a) f_1^{a(0)}(\mathbf{c}_2^a) \left( \frac{m_a}{2kT} \right)^{3/2} L_s^{3/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \times \\ & \left\{ g_2^{aa} \left[ (\mathbf{c}_1^a, \mathbf{c}_1^{a'}) L_r^{3/2} \left( \frac{m_a (c_1^{a'})^2}{2kT} \right) + (\mathbf{c}_1^a, \mathbf{c}_2^{a'}) L_r^{3/2} \left( \frac{m_a (c_2^{a'})^2}{2kT} \right) - \right. \right. \\ & \quad \left. \left( \mathbf{c}_1^a, \mathbf{c}_1^a \right) L_r^{3/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) - (\mathbf{c}_1^a, \mathbf{c}_2^a) L_r^{3/2} \left( \frac{m_a (c_2^a)^2}{2kT} \right) \right] + \\ & \quad \left( \mathbf{c}_1^a, \mathbf{c}_1^{a*} \right) L_r^{3/2} \left( \frac{m_a (c_1^{a*})^2}{2kT} \right) + (\mathbf{c}_1^a, \mathbf{c}_2^{a*}) L_r^{3/2} \left( \frac{m_a (c_2^{a*})^2}{2kT} \right) - \\ & \quad \left. \left( \mathbf{c}_1^a, \mathbf{c}_1^a \right) L_r^{3/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) - (\mathbf{c}_1^a, \mathbf{c}_2^a) L_r^{3/2} \left( \frac{m_a (c_2^a)^2}{2kT} \right) \right\}, \\ \alpha_{sr}^{(ab,a)} = & \int d\mathbf{v}_1^a d\mathbf{v}_2^b g^{ab} d\omega f_1^{a(0)}(\mathbf{c}_1^a) f_1^{b(0)}(\mathbf{c}_2^b) \left( \frac{m_a}{2kT} \right)^{3/2} L_s^{3/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \times \\ & \left\{ g_2^{ab} \left[ (\mathbf{c}_1^a, \mathbf{c}_1^{a'}) L_r^{3/2} \left( \frac{m_a (c_1^{a'})^2}{2kT} \right) - (\mathbf{c}_1^a, \mathbf{c}_1^a) L_r^{3/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \right] + \right. \\ & \quad \left. (\mathbf{c}_1^a, \mathbf{c}_1^{a*}) L_r^{3/2} \left( \frac{m_a (c_1^{a*})^2}{2kT} \right) - (\mathbf{c}_1^a, \mathbf{c}_1^a) L_r^{3/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \right\}, \\ \alpha_{sr}^{(ab,b)} = & \int d\mathbf{v}_1^a d\mathbf{v}_2^b g^{ab} d\omega f_1^{a(0)}(\mathbf{c}_1^a) f_1^{b(0)}(\mathbf{c}_2^b) \frac{m_a \sqrt{m_b}}{(2kT)^{3/2}} L_s^{3/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \times \\ & \left\{ g_2^{ab} \left[ (\mathbf{c}_1^a, \mathbf{c}_2^{b'}) L_r^{3/2} \left( \frac{m_b (c_2^{b'})^2}{2kT} \right) - (\mathbf{c}_1^a, \mathbf{c}_2^b) L_r^{3/2} \left( \frac{m_b (c_2^b)^2}{2kT} \right) \right] + \right. \\ & \quad \left. (\mathbf{c}_1^a, \mathbf{c}_2^{b*}) L_r^{3/2} \left( \frac{m_b (c_2^{b*})^2}{2kT} \right) - (\mathbf{c}_1^a, \mathbf{c}_2^b) L_r^{3/2} \left( \frac{m_b (c_2^b)^2}{2kT} \right) \right\}, \end{aligned}$$

$$-\beta_s^{(a)} = \int d\mathbf{v}_1^a f_1^{a(0)}(\mathbf{c}_1^a) \frac{m_a (c_1^a)^2}{2kT} L_s^{3/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \times$$

$$\left\{ \frac{5}{2} + \frac{m_a (c_1^a)^2}{2kT} \left[ 1 + \frac{2\pi}{5m_a} \sum_{\alpha=1}^M m_\alpha n_\alpha \sigma_{a\alpha} g_2^{a\alpha} \right] + \frac{\pi}{3} \sum_{\alpha=1}^M n_\alpha \sigma_{a\alpha}^3 g_2^{a\alpha} + \frac{2\pi m_a}{3\rho} \sum_{\alpha,\beta=1}^M n_\alpha n_\beta \sigma_{\alpha\beta}^3 g_2^{\alpha\beta} \right\}.$$

Let us consider the case, when indices  $r$  and  $s$  take two values only: zero and unity, which correspond to the first polynomial approximation. In such a case the system of equations becomes finite. It is convenient to represent it in the matrix form:

$$\hat{a}_{ij} \hat{A}_j = \hat{b}_i,$$

$$\hat{A}_j = \text{col} \left( A_0^a, A_1^a, A_0^b, A_1^b \right), \quad \hat{b}_i = \text{col} \left( \beta_0^{(a)}, \beta_1^{(a)}, \beta_0^{(b)}, \beta_1^{(b)} \right),$$

matrix  $\hat{a}_{ij}$  reads

$$\begin{bmatrix} \alpha_{00}^{(ab,a)} = a_{11} & \alpha_{01}^{(ab,a)} = a_{12} & \alpha_{00}^{(ab,b)} = a_{13} & \alpha_{01}^{(ab,b)} = a_{14} \\ \alpha_{10}^{(ab,a)} = a_{21} & \alpha_{11}^{(ab,a)} + \alpha_{11}^{(a)} = a_{22} & \alpha_{10}^{(ab,b)} = a_{23} & \alpha_{11}^{(ab,b)} = a_{24} \\ \alpha_{00}^{(ba,a)} = a_{31} & \alpha_{01}^{(ba,a)} = a_{32} & \alpha_{00}^{(ba,b)} = a_{33} & \alpha_{01}^{(ba,b)} = a_{34} \\ \alpha_{10}^{(ba,a)} = a_{41} & \alpha_{11}^{(ba,a)} = a_{42} & \alpha_{10}^{(ba,b)} = a_{43} & \alpha_{11}^{(ba,b)} + \alpha_{11}^{(b)} = a_{44} \end{bmatrix}.$$

The solution is written using the Kramers' method at once:

$$A_0^a = \frac{1}{\det \hat{a}_{ij}} \begin{vmatrix} b_1 & a_{12} & a_{13} & a_{14} \\ b_2 & a_{22} & a_{23} & a_{24} \\ b_3 & a_{32} & a_{33} & a_{34} \\ b_4 & a_{42} & a_{43} & a_{44} \end{vmatrix}, \quad A_1^a = \frac{1}{\det \hat{a}_{ij}} \begin{vmatrix} a_{11} & b_1 & a_{13} & a_{14} \\ a_{21} & b_2 & a_{23} & a_{24} \\ a_{31} & b_3 & a_{33} & a_{34} \\ a_{41} & b_4 & a_{43} & a_{44} \end{vmatrix},$$

$$A_0^b = \frac{1}{\det \hat{a}_{ij}} \begin{vmatrix} a_{11} & a_{12} & b_1 & a_{14} \\ a_{21} & a_{22} & b_2 & a_{24} \\ a_{31} & a_{32} & b_3 & a_{34} \\ a_{41} & a_{42} & b_4 & a_{44} \end{vmatrix}, \quad A_1^b = \frac{1}{\det \hat{a}_{ij}} \begin{vmatrix} a_{11} & a_{11} & a_{13} & b_1 \\ a_{21} & a_{21} & a_{23} & b_2 \\ a_{31} & a_{31} & a_{33} & b_3 \\ a_{41} & a_{41} & a_{43} & b_4 \end{vmatrix}.$$

Quantities  $a_{ij}$  can be calculated and represented via so-called  $\Omega$ -integrals (see Appendix C.). Then one obtains:

$$a_{11} = -n_a n_b \sqrt{\frac{32}{\pi}} \frac{m^*}{m_a} \left( g_2^{ab}(\sigma_{ab}|n, \beta) {}^{aa}\Omega_{\text{hs}}^{(1,1)} + {}^{aa}\Omega_1^{(1,1)} \right),$$

$$a_{12} = a_{21} = -n_a n_b \sqrt{\frac{32}{\pi}} \left( \frac{m^*}{m_a} \right)^2 \times$$

$$\left( g_2^{ab}(\sigma_{ab}|n, \beta) \left[ \frac{5}{2} {}^{aa}\Omega_{\text{hs}}^{(1,1)} - {}^{aa}\Omega_{\text{hs}}^{(1,2)} \right] + \frac{5}{2} {}^{aa}\Omega_1^{(1,1)} - {}^{aa}\Omega_1^{(1,2)} \right),$$

$$a_{13} = 4n_a n_b \sqrt{\frac{m^*}{\pi m_b}} \left( g_2^{ab}(\sigma_{ab}|n, \beta) {}^{ab}\Omega_{\text{hs}}^{(1,1)} + {}^{ab}\Omega_1^{(1,1)} \right),$$

$$\begin{aligned}
a_{14} &= a_{23} = 4n_a n_b \sqrt{\pi} \left( \frac{m^*}{m_b} \right)^{3/2} \times \\
&\left( g_2^{ab}(\sigma_{ab}|n, \beta) \left[ \frac{5}{2} {}^{ab}\Omega_{\text{hs}}^{(1,1)} - {}^{ab}\Omega_{\text{hs}}^{(1,2)} \right] + \frac{5}{2} {}^{ab}\Omega_1^{(1,1)} - {}^{ab}\Omega_1^{(1,2)} \right), \\
\\
a_{22} &= -n_a n_b \sqrt{\frac{32}{\pi}} \left( \frac{m^*}{m_a} \right) \times \\
&\left( g_2^{ab}(\sigma_{ab}|n, \beta) \left[ \left\{ \frac{30}{4} \left( \frac{m^*}{m_b} \right)^2 + \frac{25}{4} \left( \frac{m^*}{m_a} \right)^2 \right\} {}^{aa}\Omega_{\text{hs}}^{(1,1)} - \right. \right. \\
&5 \left( \frac{m^*}{m_a} \right)^2 {}^{aa}\Omega_{\text{hs}}^{(1,2)} + \left( \frac{m^*}{m_a} \right)^2 {}^{aa}\Omega_{\text{hs}}^{(1,3)} + \frac{2m_a m_b}{(m_a + m_b)^2} {}^{aa}\Omega_{\text{hs}}^{(2,2)} \left. \right] + \\
&\left[ \left\{ \frac{30}{4} \left( \frac{m^*}{m_b} \right)^2 + \frac{25}{4} \left( \frac{m^*}{m_a} \right)^2 \right\} {}^{aa}\Omega_1^{(1,1)} - \right. \\
&5 \left( \frac{m^*}{m_a} \right)^2 {}^{aa}\Omega_1^{(1,2)} + \left( \frac{m^*}{m_a} \right)^2 {}^{aa}\Omega_1^{(1,3)} + \frac{2m_a m_b}{(m_a + m_b)^2} {}^{aa}\Omega_1^{(2,2)} \left. \right] \Bigg) + \\
&n_a^2 \sqrt{\frac{8}{\pi}} \left( g_2^{aa}(\sigma_{ab}|n, \beta) {}^{aa}\Omega_{\text{hs}}^{(2,2)} + {}^{aa}\Omega_1^{(2,2)} \right), \\
\\
a_{24} &= 4n_a n_b \frac{1}{\sqrt{\pi}} \frac{(m^*)^{5/2}}{m_a m_b^{3/2}} \times \\
&\left( g_2^{ab}(\sigma_{ab}|n, \beta) \left( \frac{55}{4} {}^{ab}\Omega_{\text{hs}}^{(1,1)} - 5 {}^{ab}\Omega_{\text{hs}}^{(1,2)} + {}^{ab}\Omega_{\text{hs}}^{(1,3)} - 2 {}^{ab}\Omega_{\text{hs}}^{(2,2)} \right) + \right. \\
&\left. \frac{55}{4} {}^{ab}\Omega_1^{(1,1)} - 5 {}^{ab}\Omega_1^{(1,2)} + {}^{ab}\Omega_1^{(1,3)} - 2 {}^{ab}\Omega_1^{(2,2)} \right).
\end{aligned}$$

The remaining quantities  $a_{ij}$  are expressed via previous ones by simple exchange of indices  $a \rightarrow b$  and vice versa,  $b \rightarrow a$ :

$$\begin{aligned}
a_{31} &= a_{13} & | & \quad a \rightarrow b, b \rightarrow a, \\
a_{32} &= a_{41} = a_{14} & | & \quad a \rightarrow b, b \rightarrow a, \\
a_{33} &= a_{11} & | & \quad a \rightarrow b, b \rightarrow a, \\
a_{34} &= a_{43} = a_{12} & | & \quad a \rightarrow b, b \rightarrow a, \\
a_{42} &= a_{24} & | & \quad a \rightarrow b, b \rightarrow a, \\
a_{44} &= a_{22} & | & \quad a \rightarrow b, b \rightarrow a.
\end{aligned}$$

The matrix elements  $b_i$  in the first polynomial approximation are calculated exactly:

$$b_1 = \frac{5}{4} n_a \left( 1 + \frac{2\pi}{5m_a} \sum_{\alpha=1}^s m_{\alpha} n_{\alpha} \sigma_{a\alpha}^3 g_2^{a\alpha}(\sigma_{a\alpha}|n, \beta) \right) -$$

$$\begin{aligned}
& \frac{n_a}{2} \left( \frac{5}{2} + \frac{\pi}{3} \sum_{\alpha=1}^s n_{\alpha} \sigma_{a\alpha}^3 g_2^{a\alpha}(\sigma_{a\alpha}|n, \beta) + \right. \\
& \left. \frac{2\pi m_a}{3\rho} \sum_{\alpha, \beta=1}^s n_{\alpha} n_{\beta} \sigma_{\alpha\beta}^3 g_2^{\alpha\beta}(\sigma_{\alpha\beta}|n, \beta) \right), \\
b_2 &= -\frac{15}{4} n_a \left( 1 + \frac{2\pi}{5m_a} \sum_{\alpha=1}^s m_{\alpha} n_{\alpha} \sigma_{a\alpha}^3 g_2^{a\alpha}(\sigma_{a\alpha}|n, \beta) \right),
\end{aligned}$$

$$\begin{aligned}
b_3 &= b_1 & | & \quad a \rightarrow b, b \rightarrow a, \\
b_4 &= b_2 & | & \quad a \rightarrow b, b \rightarrow a,
\end{aligned}$$

$$\text{where } \rho = \sum_{\alpha=1}^s \rho_{\alpha} = \sum_{\alpha=1}^s m_{\alpha} n_{\alpha}.$$



## Appendix B. Calculation of $B_n^\alpha$ -coefficients

The two mutually exchangeable infinite systems of equations for searching  $B_n^\alpha$ -coefficients could be written in the form

$$\sum_{r=0}^{\infty} \left[ \left( \gamma_{sr}^{(a)} + \gamma_{sr}^{(ab,a)} \right) B_r^a + \gamma_{sr}^{(ab,b)} B_r^b \right] = \zeta_s^{(a)},$$

$$\sum_{r=0}^{\infty} \left[ \gamma_{sr}^{(ba,a)} B_r^a + \left( \gamma_{sr}^{(b)} + \gamma_{sr}^{(ba,b)} \right) B_r^b \right] = \zeta_s^{(b)},$$

where quantities  $\gamma_{sr}$ ,  $\zeta_s$  have an integral structure and are expressed via  $L_n^r$  polynomials. General expressions for these quantities are very cumbersome and their forms depend on the structure of collision integrals. But general form is always the same and is completely shown in [28]. In our case

$$\begin{aligned} \gamma_{sr}^{(a)} &= \int d\mathbf{v}_1^a d\mathbf{v}_2^a g^{aa} d\omega f_1^{a(0)}(c_1^a) f_1^{a(0)}(c_2^a) \left( \frac{m_a}{2kT} \right)^2 L_s^{5/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) c_1^a c_1^a \times \\ &\left\{ g_2^{aa} \left[ L_r^{5/2} \left( \frac{m_a (c_1^{a'})^2}{2kT} \right) \left( c_1^{a'} c_1^{a'} - \frac{1}{3} (c_1^{a'})^2 \overleftrightarrow{I} \right) + L_r^{5/2} \left( \frac{m_a (c_2^{a'})^2}{2kT} \right) \left( c_2^{a'} c_2^{a'} - \frac{1}{3} (c_2^{a'})^2 \overleftrightarrow{I} \right) - \right. \right. \\ &\left. L_r^{5/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \left( c_1^a c_1^a - \frac{1}{3} (c_1^a)^2 \overleftrightarrow{I} \right) - L_r^{5/2} \left( \frac{m_a (c_2^a)^2}{2kT} \right) \left( c_2^a c_2^a - \frac{1}{3} (c_2^a)^2 \overleftrightarrow{I} \right) \right] + \\ &\left. L_r^{5/2} \left( \frac{m_a (c_1^{a*})^2}{2kT} \right) \left( c_1^{a*} c_1^{a*} - \frac{1}{3} (c_1^{a*})^2 \overleftrightarrow{I} \right) + L_r^{5/2} \left( \frac{m_a (c_2^{a*})^2}{2kT} \right) \left( c_2^{a*} c_2^{a*} - \frac{1}{3} (c_2^{a*})^2 \overleftrightarrow{I} \right) - \right. \\ &\left. L_r^{5/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \left( c_1^a c_1^a - \frac{1}{3} (c_1^a)^2 \overleftrightarrow{I} \right) - L_r^{5/2} \left( \frac{m_a (c_2^a)^2}{2kT} \right) \left( c_2^a c_2^a - \frac{1}{3} (c_2^a)^2 \overleftrightarrow{I} \right) \right\}, \\ \gamma_{sr}^{(ab,a)} &= \int d\mathbf{v}_1^a d\mathbf{v}_2^a g^{ab} d\omega f_1^{a(0)}(c_1^a) f_1^{b(0)}(c_2^b) \left( \frac{m_a}{2kT} \right)^2 L_s^{5/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) c_1^a c_1^a \times \\ &\left\{ g_2^{ab} \left[ L_r^{5/2} \left( \frac{m_a (c_1^{a'})^2}{2kT} \right) \left( c_1^{a'} c_1^{a'} - \frac{1}{3} (c_1^{a'})^2 \overleftrightarrow{I} \right) - L_r^{5/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \left( c_1^a c_1^a - \frac{1}{3} (c_1^a)^2 \overleftrightarrow{I} \right) \right] + \right. \\ &\left. L_r^{5/2} \left( \frac{m_a (c_1^{a*})^2}{2kT} \right) \left( c_1^{a*} c_1^{a*} - \frac{1}{3} (c_1^{a*})^2 \overleftrightarrow{I} \right) - L_r^{5/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \left( c_1^a c_1^a - \frac{1}{3} (c_1^a)^2 \overleftrightarrow{I} \right) \right\}, \\ \gamma_{sr}^{(ab,b)} &= \int d\mathbf{v}_1^a d\mathbf{v}_2^a g^{ab} d\omega f_1^{a(0)}(c_1^a) f_1^{b(0)}(c_2^b) \frac{m_a m_b}{(2kT)^2} L_s^{5/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) c_1^a c_1^a \times \\ &\left\{ g_2^{ab} \left[ L_r^{5/2} \left( \frac{m_b (c_2^{b'})^2}{2kT} \right) \left( c_2^{b'} c_2^{b'} - \frac{1}{3} (c_2^{b'})^2 \overleftrightarrow{I} \right) - L_r^{5/2} \left( \frac{m_b (c_2^b)^2}{2kT} \right) \left( c_2^b c_2^b - \frac{1}{3} (c_2^b)^2 \overleftrightarrow{I} \right) \right] + \right. \\ &\left. L_r^{5/2} \left( \frac{m_b (c_2^{b*})^2}{2kT} \right) \left( c_2^{b*} c_2^{b*} - \frac{1}{3} (c_2^{b*})^2 \overleftrightarrow{I} \right) - L_r^{5/2} \left( \frac{m_b (c_2^b)^2}{2kT} \right) \left( c_2^b c_2^b - \frac{1}{3} (c_2^b)^2 \overleftrightarrow{I} \right) \right\}, \end{aligned}$$

$$\begin{aligned} \zeta_s^{(a)} = & - \int d\mathbf{v}_1^a f_1^{a(0)}(\mathbf{c}_1^a) L_s^{5/2} \left( \frac{m_a (c_1^a)^2}{2kT} \right) \mathbf{c}_1^a \mathbf{c}_1^a \times \\ & \left\{ \frac{m_a (c_1^a)^2}{3kT} \left( 1 + \frac{2\pi}{3n} \sum_{\alpha, \beta=1}^M n_\alpha n_\beta \sigma_{\alpha\beta}^3 g_2^{\alpha\beta} - \frac{2\pi}{5m_a} \sum_{\alpha=1}^M m_\alpha n_\alpha \sigma_{a\alpha}^3 g_2^{a\alpha} \right) \vec{I} + \right. \\ & \left. \frac{2\pi}{3} \sum_{\alpha=1}^M \left[ n_\alpha \sigma_{a\alpha}^3 g_2^{a\alpha} - \sum_{\beta=1}^M \frac{n_\alpha n_\beta}{n} \sigma_{\alpha\beta}^3 g_2^{\alpha\beta} \right] \vec{I} - \frac{m_a}{kT} \mathbf{c}_1^a \mathbf{c}_1^a \left[ 1 + \frac{4\pi}{15m_a} \sum_{\alpha=1}^M m_\alpha n_\alpha \sigma_{a\alpha}^3 g_2^{a\alpha} \right] \right\}. \end{aligned}$$

Let us consider the case, when indices  $r$  and  $s$  take one zero value only, which corresponds to the zeroth polynomial approximation. In such a case the system of equations becomes finite. It is convenient to represent it in the matrix form:

$$\begin{aligned} \hat{c}_{ij} \hat{B}_j &= \hat{d}_i, \\ \hat{B}_j &= \text{col} \left( B_0^a, B_0^b \right), \quad \hat{d}_i = \text{col} \left( \zeta_0^{(a)}, \zeta_0^{(b)} \right), \end{aligned}$$

matrix  $\hat{c}_{ij}$  reads

$$\begin{pmatrix} \gamma_{00}^{(ab,a)} + \gamma_{00}^{(a)} & = & c_{11} & \gamma_{00}^{(ab,b)} & = & c_{12} \\ \gamma_{00}^{(ba,a)} & = & c_{21} & \gamma_{00}^{(ba,b)} + \gamma_{00}^{(b)} & = & c_{22} \end{pmatrix}.$$

The solution is written using the Kramers' method at once:

$$B_0^a = \frac{1}{\det \hat{c}_{ij}} \begin{vmatrix} d_1 & c_{12} \\ d_2 & c_{22} \end{vmatrix}, \quad B_0^b = \frac{1}{\det \hat{c}_{ij}} \begin{vmatrix} c_{11} & d_1 \\ c_{21} & d_2 \end{vmatrix}.$$

Quantities  $c_{ij}$  can be calculated and represented via so-called  $\Omega$ -integrals (see Appendix C.). Then one obtains:

$$\begin{aligned} c_{11} = & -n_a n_b \frac{8}{3} \sqrt{\frac{kT}{\pi m_a}} \frac{(m^*)^2}{m_a m_b} \times \\ & \left( g_2^{ab}(\sigma_{ab}|n, \beta) \left[ 5^{aa} \Omega_{\text{hs}}^{(1,1)} + \frac{3m_b}{2m_a} \Omega_{\text{hs}}^{(2,2)} \right] + 5^{aa} \Omega_1^{(1,1)} + \frac{3m_b}{2m_a} \Omega_1^{(2,2)} \right) \\ & + 4n_a^2 \sqrt{\frac{kT}{\pi m_a}} \left( g_2^{aa}(\sigma_{ab}|n, \beta) \Omega_{\text{hs}}^{(2,2)} + {}^{aa} \Omega_1^{(2,2)} \right), \\ c_{12} = & \frac{8n_a n_b \sqrt{2kTm^*}}{3(m_a + m_b)} \times \\ & \left( g_2^{ab}(\sigma_{ab}|n, \beta) \left[ 5^{ab} \Omega_{\text{hs}}^{(1,1)} - \frac{3}{2} {}^{ab} \Omega_{\text{hs}}^{(2,2)} \right] + 5^{ab} \Omega_1^{(1,1)} - \frac{3}{2} {}^{ab} \Omega_1^{(2,2)} \right). \end{aligned}$$

The remaining quantities  $c_{ij}$  are expressed via previous ones by simple exchange of indices  $a \rightarrow b$  and vice versa,  $b \rightarrow a$ :

$$\begin{aligned} c_{21} = c_{12} & \quad | \quad a \rightarrow b, b \rightarrow a, \\ c_{22} = c_{11} & \quad | \quad a \rightarrow b, b \rightarrow a. \end{aligned}$$

The matrix elements  $d_i$  in the zeroth polynomial approximation are calculated exactly:

$$\begin{aligned}
d_1 = & n_a \frac{5}{2} \left( 1 + \frac{2\pi}{3n} \sum_{\alpha, \beta=1}^s n_\alpha n_\beta \sigma_{\alpha\beta}^3 g_2^{\alpha\beta}(\sigma_{\alpha\beta}|n, \beta) - \right. \\
& \left. \frac{2\pi}{3m_a} \sum_{\alpha=1}^s m_\alpha n_\alpha \sigma_{a\alpha}^3 g_2^{a\alpha}(\sigma_{a\alpha}|n, \beta) \right) - \\
& \frac{\pi n_a}{3} \sum_{\alpha=1}^s \left[ n_\alpha \sigma_{a\alpha}^3 g_2^{a\alpha}(\sigma_{a\alpha}|n, \beta) - \sum_{\beta=1}^s \frac{n_\alpha n_\beta}{n} \sigma_{\alpha\beta}^3 g_2^{\alpha\beta}(\sigma_{\alpha\beta}|n, \beta) \right] \\
& + \frac{117\pi n_a}{56} \left( 1 + \frac{4\pi}{15m_a} \sum_{\alpha=1}^s m_\alpha n_\alpha \sigma_{a\alpha}^3 g_2^{a\alpha}(\sigma_{a\alpha}|n, \beta) \right), \\
d_2 = & d_1 \quad | \quad a \rightarrow b, b \rightarrow a.
\end{aligned}$$

## Appendix C. $\Omega$ -integrals

To calculate  $\Omega$ -integrals analytically, one should find first the expressions for scattering angles  $\chi'$  and  $\chi^*$ . In general case both these angles are complicated functions of many factors:

$$\chi'^{*} = \pi - 2b \int_{r_0}^{\infty} dr r^2 \left( 1 - \frac{b}{r^2} - \frac{2\Phi^{\text{hs},1}(r)}{m^* g^2} \right)^{-1/2},$$

here the meaning of  $r_0$  is searched from the extremum condition

$$r_0^2 - b^2 - \frac{2r_0^2}{m^* g^2} \Phi^{\text{hs},1}(r_0) = 0.$$

In view of geometrical reasoning, one can calculate  $\cos(\chi')$  exactly [30] (Figure C1):

$$\cos(\chi'/2) = b/\sigma_{\alpha\beta}.$$

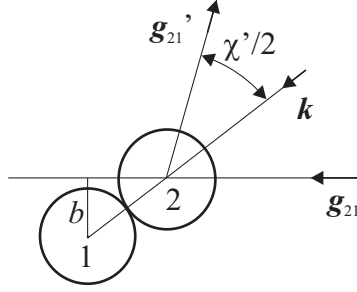


Figure C1: Collision of hard spheres.

In the paper [26],  $\cos(\chi^*)$  is calculated approximately. In small scattering angles approximation one may be thought of as  $|\mathbf{g}'| \approx |\mathbf{g}|$  and  $\Delta\mathbf{g} = \mathbf{g}_y$ . Considering the scattering dynamics as is shown in the drawing below (see Figure C2), one may express  $\cos(\chi^*) = \sin\theta$  ( $\chi^* + \Theta = \pi/2$ ) via  $\Delta\mathbf{g}$  and make expansion in the Teilor series. Here  $\mathbf{F}$  denotes the Coulomb force, which acts on particle 2 from the side of particle 1; in considered reference frame particle 1 is fixed while particle 2 impacts from infinity. Hence taking into account (3.7) one may obtain:

$$\cos(\chi^*) = 1 - \frac{1}{2} \left( \frac{Z_\alpha Z_\beta e^2 \pi}{2kTby^2} \right)^2.$$

This allows to write all  $\Omega$ -integrals in an analytical form, which appear at searching  $A_n^\alpha$ -coefficients (see Appendix A.),  $B_n^\alpha$ -coefficients (see Appendix B.) and

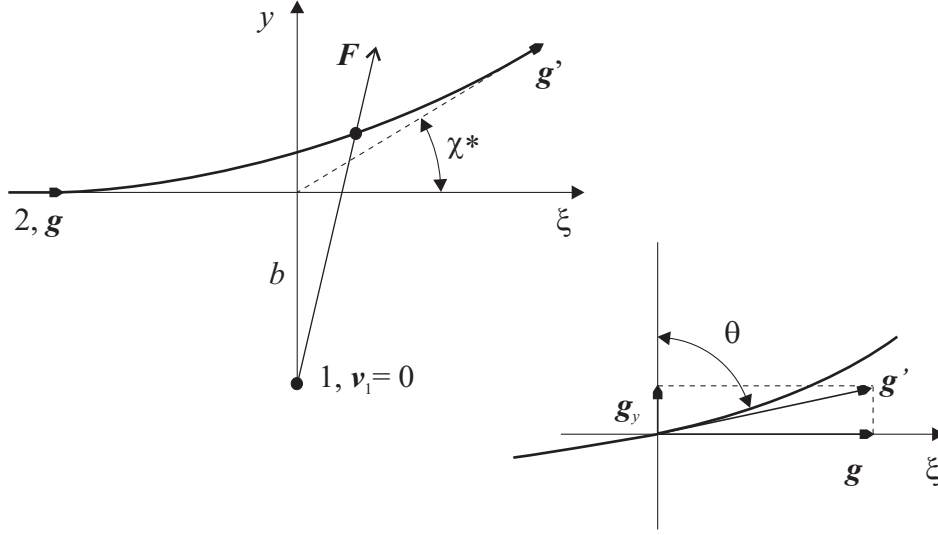


Figure C2: Scattering of two charged particles.

$E_n^\alpha$ -coefficients:

$$\begin{aligned}
 \alpha\beta \Omega_{\text{hs}}^{(1,1)} &= \pi \sigma_{\alpha\beta}^2, & \alpha\beta \Omega_1^{(1,1)} &= \frac{\pi^3}{2} \left( \frac{Z_\alpha Z_\beta e^2}{2kT} \right)^2 \ln \frac{D}{\sigma_{\alpha\beta}}, \\
 \alpha\beta \Omega_{\text{hs}}^{(1,2)} &= 3 \left[ \alpha\beta \Omega_{\text{hs}}^{(1,1)} \right], & \alpha\beta \Omega_1^{(1,2)} &= 2 \left[ \alpha\beta \Omega_1^{(1,1)} \right], \\
 \alpha\beta \Omega_{\text{hs}}^{(1,3)} &= 12 \left[ \alpha\beta \Omega_{\text{hs}}^{(1,1)} \right], & \alpha\beta \Omega_1^{(1,3)} &= 12 \left[ \alpha\beta \Omega_1^{(1,1)} \right], \\
 \alpha\beta \Omega_{\text{hs}}^{(2,2)} &= 2 \left[ \alpha\beta \Omega_{\text{hs}}^{(1,1)} \right], & \alpha\beta \Omega_1^{(2,2)} &= 4 \left[ \alpha\beta \Omega_1^{(1,1)} \right],
 \end{aligned}$$

where the new quantity  $D$  has been introduced, while calculating  $\alpha\beta \Omega_1^{(r,p)}$ . It is the cut-off radius of upper limit in the integral with respect to  $b$ . This is needed to avoid the difficulty of logarithmical divergency in  $\Omega_l$ -integrals at long distances. The quantity  $D$  has dimensionality of distance and meaning of screening radius (like a Debye radius). The equation for obtaining  $D$  in any explicit form is a separate task, which will be considered in a next paper.

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## Figure 1 caption

The temperature dependencies of some transport coefficients for binary mixtures: a), c) and e) for neutral systems at  $\Delta = 0.1$ ; b), d) and f) for once-ionized gases at  $\Delta = 0.0125$ . Everywhere  $T$ : [K]. a), b) are mutual diffusion coefficients for mixtures of Ar-Kr ( $\bullet$ ), Ar-Xe ( $\blacksquare$ ), Kr-Xe ( $\blacktriangle$ ),  $D^{\alpha\beta}$  a):  $10^{-7}$ , b):  $10^{-9}$ [m<sup>2</sup>/s]. c), d) are thermal diffusion coefficients for Ar-Kr mixture: argon's Ar (+) and krypton's Kr ( $\blacklozenge$ ) components,  $D_T$  c):  $10^{-5}$ , d):  $10^{-8}$ [kg/m·s]. e), f) are shear viscosity coefficients for Ar-Kr mixture ( $\bullet$ ), argon's Ar ( $\blacksquare$ ) and krypton's Kr ( $\blacktriangle$ ) components,  $\eta$  e):  $10^{-4}$ , f):  $10^{-6}$ [Pa·s].



